Signal Analysis & Communication ECE355

Ch. 3.3: Fourier Series Representation of CT Periodic Signals

Lecture 14

11 - 10 - 2023

Ch. 3·3: CT FOURIER SERIES

- **Recall:**

1. Periodic signal $x_{(t)}$: $x_{(t)} = x_{(t+T)}, \ \forall t$
2. Fundamental period: minimum positive value of `T'
3. Fundamental angular freq.: $\omega_0 = \frac{2\pi}{T}$

FACT $\underbrace{\text{"Almost all"}}_{\text{we will see later}}$ periodic signals can be expressed as a sum of harmonically related complex exponentials.

$$\Rightarrow \quad x_{(t)} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

here, $a_k e^{jk\omega_0 t}$: $|k|$th harmonic component for $k \neq 0$.

($k = \pm 1$: first harmonic components "fundamental components"

$k = 0 \Rightarrow a_0$: constant dc component)

Example

$$x_{(t)} = 1 + \frac{1}{2}\cos(2\pi t) + \cos(4\pi t) + \frac{2}{3}\cos(6\pi t)$$

Fundamental period $T = 1$.

Frequency $\omega_0 = 2\pi$

- Can be expressed as sum of exponents.

$$x_{(t)} = 1 + \frac{1}{4}\left(e^{j2\pi t} + e^{-j2\pi t}\right) + \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right) + \frac{1}{3}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$x_{(t)} = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt}$$

where

$$a_0 = 1$$

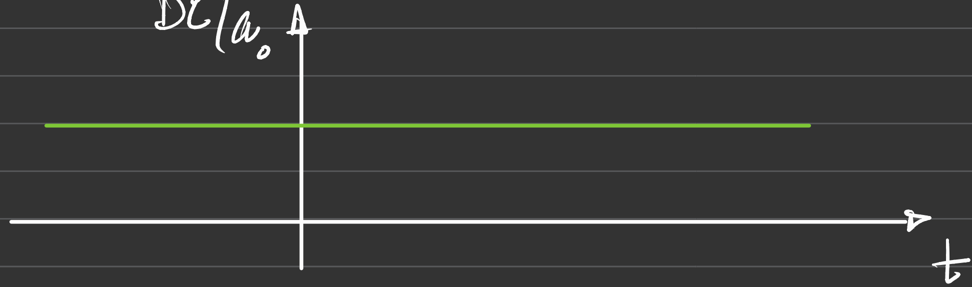
$$a_1 = a_{-1} = 1/4$$

$$a_2 = a_{-2} = 1/2$$

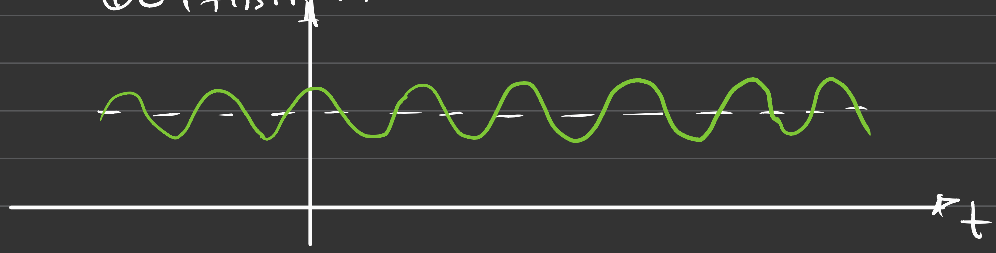
$$a_3 = a_{-3} = 1/3$$

$$a_k = 0 \quad , \quad o/w.$$

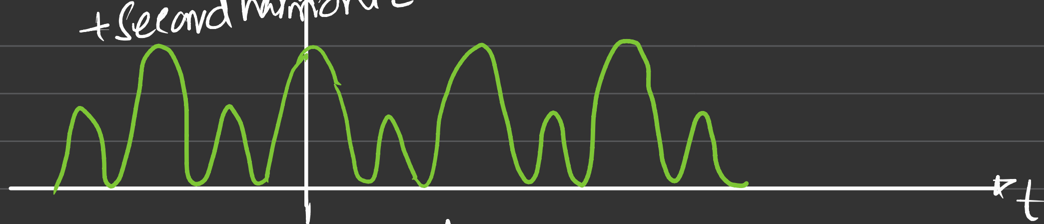
DC/a_0



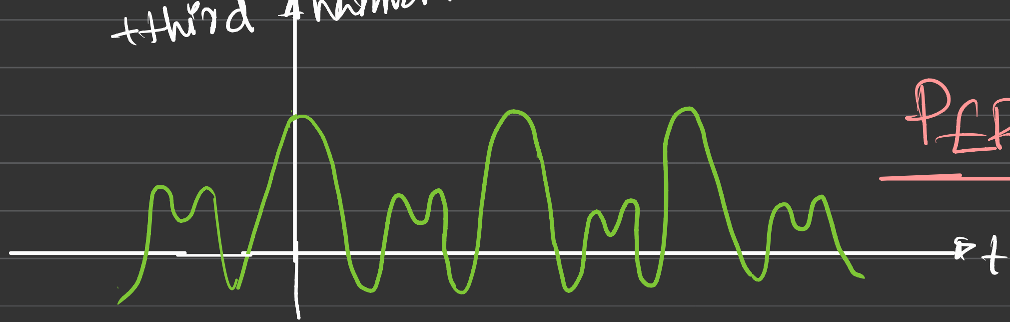
DC + first harmonic



DC + first harmonic + second harmonic



DC + first + second + third harmonics



PERIODIC

NOTE: LTI:

$$x_{(t)} \longrightarrow \boxed{h_{(t)}} \longrightarrow y_{(t)}$$

- Express I/P as

$$x_{(t)} = \sum_k a_k e^{jk\omega_0 t}$$

Linear combination of complex exponentials

- The O/P is given as

$$y_{(t)} = \sum_k a_k H_{(jk\omega_0)} e^{jk\omega_0 t}$$

- Here,

$H_{(jk\omega_0)}$ / $H_{(j\omega)}$ is called the "Frequency Response" of LTI sys.

For CT periodic signals.

We will see later for CT aperiodic signals.

+ Review of the content for Midterm Exam 1.
